



Afstanden en hoeken in de ruimte

Oplossing oefening 1

Geg.: Kubus $\begin{pmatrix} EFGH \\ ABCD \end{pmatrix}$

$$|AB| = 8$$

$$P \in [GH], |PH| = |PG|$$

Gevr.: $A \hat{P} C$

Opl.:

- Berekening $|AH|$ met stelling van Pythagoras in $\triangle AEH$

$$\begin{aligned} |AH|^2 &= |AE|^2 + |EH|^2 &\Rightarrow |AH|^2 &= 8^2 + 8^2 = 64 + 64 = 128 \\ & &\Rightarrow |AH| &= \sqrt{128} \end{aligned}$$

- Daarna: berekening $|AP|$ met stelling van Pythagoras in $\triangle AHP$

$$\begin{aligned} |AP|^2 &= |AH|^2 + |HP|^2 &\Rightarrow |AP|^2 &= (\sqrt{128})^2 + 4^2 = 128 + 16 = 144 \\ & &\Rightarrow |AP| &= \sqrt{144} = 12 \end{aligned}$$

- Analoog: berekening $|CP|$ met stelling van Pythagoras in $\triangle CGP$

$$\begin{aligned} |CP|^2 &= |CG|^2 + |GP|^2 &\Rightarrow |CP|^2 &= 8^2 + 4^2 = 64 + 16 = 80 \\ & &\Rightarrow |CP| &= \sqrt{80} \end{aligned}$$

- Analoog: berekening $|AC|$ met stelling van Pythagoras in $\triangle ABC$

$$\begin{aligned} |AC|^2 &= |AB|^2 + |BC|^2 &\Rightarrow |AC|^2 &= 8^2 + 8^2 = 64 + 64 = 128 \\ & &\Rightarrow |AC| &= \sqrt{128} \end{aligned}$$

- Nu kunnen we $A \hat{P} C$ berekenen met de cosinusregel in $\triangle APC$:

$$|AC|^2 = |AP|^2 + |CP|^2 - 2 \cdot |AP| \cdot |CP| \cdot \cos(A \hat{P} C) \quad \Rightarrow \quad \cos(A \hat{P} C) = \frac{|AC|^2 - |AP|^2 - |CP|^2}{-2 \cdot |AP| \cdot |CP|}$$

$$\text{Dus } \cos(A \hat{P} C) = \frac{128 - 144 - 80}{-2 \cdot 12 \cdot \sqrt{80}} = 0,44721 \quad \Rightarrow \quad A \hat{P} C = 63^\circ 26' 6''$$

